

John Brillhart. At the time of completion of these results we learned of similar work† by Donald B. Gillies on Illiac II. We compared our residues with his and found ten discrepancies. A check revealed that one of our three supposedly identical program decks contained an error. The questionable residues were recalculated and found to agree with Dr. Gillies' values. These residues are marked by an asterisk (*).

The authors have verified that Riesel's M_{3217} and Hurwitz's M_{4253} and M_{4423} are prime. Hurwitz's octal remainder [1] of 72013 for the prime exponent 3301 was also verified. The running time for p near 6500 was three hours, using an IBM 7090.

Picatinny Arsenal
Dover, New Jersey
Standard Oil Company of California
San Francisco, California

1. A. HURWITZ, "New Mersenne primes", *Math. Comp.*, v. 16, 1962, p. 249-51.
2. A. HURWITZ, Private communication to the authors dated March 12, 1962.
3. H. RIESEL, "Mersenne numbers", *MTAC*, v. 12, 1958, p. 207.
4. H. RIESEL, "All factors $q < 10^8$ in all Mersenne numbers, $2^p - 1$, p a prime $< 10^6$ ", *Math. Comp.*, v. 16, 1962, p. 478-82. Errata, *Math. Comp.*, v. 17, 1963, p. 486.
5. S. KRAVITZ & M. BERG, "Recent research in Mersenne numbers" *Recreational Mathematics Magazine*, October, 1962. p. 40.

Note on the Congruence $a^{p-1} \equiv 1 \pmod{p^2}$.

By Hans Riesel

During 1961 the author with the aid of the electronic computer BESK gathered some data concerning the residues

$$a^{p-1} \pmod{p^2},$$

for odd primes p , from which the following tables have been compiled. In the case of $p < 1000$, the residues of $a^{p-1} \pmod{p^2}$ are available for comparison. For larger p the program has printed out only those residues $\equiv 0 \pmod{p^2}$. The running time on BESK was put at the author's disposal by courtesy of the Swedish Board for Computing Machinery.

Mabarsstigen 2
Stockholm-Vallingby, Sweden

All odd primes $p < 500000$ with $a^{p-1} \equiv 1 \pmod{p^2}$ for $a \leq 10$.

a	p	a	p
2	1093, 3511	6	66161
3	11	7	5, 491531
4	1093, 3511	8	3, 1093, 3511
5	20771, 40487	9	11
		10	3, 487

All odd primes < 10000 with $a^{p-1} \equiv 1 \pmod{p^2}$ for $11 \leq a \leq 150$.

a	p	a	p	a	p
11	71	56	647	102	7559
12	2693	57	5	104	313
13	863	58	131	105	7669
14	29, 353	59	2777	107	3, 5, 97
16	1093, 3511	60	29	108	3761
17	3	62	3, 19, 127, 1291	109	3
18	5, 7, 37, 331	63	23, 29	110	17, 5381, 9431
19	3, 7, 13, 43, 137	64	3, 1093, 3511	111	131
20	281	65	17, 163	112	11
22	13, 673	67	7, 47	114	9181
23	13	68	5, 7, 19, 113, 2741	115	31
24	5	69	19, 223, 631	116	3, 7, 19, 47
26	3, 5, 71	70	13	117	7, 31, 37
27	11	71	3, 47, 331	118	3, 5, 11, 23
28	3, 19, 23	73	3	119	1741
30	7	74	5	120	11, 653
31	7, 79, 6451	75	17, 43, 347	121	71
32	5, 1093, 3511	76	5, 37, 1109, 9241	122	11, 2791
33	233	78	43, 151, 181, 1163	124	5, 11
35	3, 1613, 3571	79	7, 263, 3037	125	3
37	3	80	3, 7, 13, 6343	126	5
38	17, 127	81	11	127	3, 19, 907
39	8039	82	3, 5	128	7, 1093, 3511
40	11, 17, 307	83	4871	129	7, 113
41	29	84	163, 653	130	11, 23
42	23	87	1999	131	17
43	5, 103	89	3, 13	132	5
44	3, 229, 5851	91	3, 293	134	3, 17
45	1283	92	727	136	3, 5153
46	3, 829	93	5, 509, 9221	137	29, 59, 6733
48	7, 257	94	11, 241	138	97, 4889
49	5	95	2137	143	3, 5, 67, 197, 1999
50	7	96	109, 5437, 8329	144	2693
51	5, 41	97	7	145	3, 31
52	461	98	3	146	7, 13, 79
53	3, 47, 59, 97	99	5, 7, 13, 19, 83	147	13, 79, 103, 283
54	19, 1949	100	3, 487	148	7, 11, 41
55	3	101	5	149	5
				150	13